Energy Efficiency Optimization with Simultaneous Wireless Information and Power Transfer in MIMO Broadcast Channels

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Abstract

Simultaneous wireless information and power transfer (SWIPT) is anticipated to have great applications in fifth-generation (5G) and beyond communication systems. In this paper, we address the energy efficiency (EE) optimization problem for SWIPT multiple-input multiple-output broadcast channel (MIMO-BC) with time-switching (TS) receiver design. Our aim is to maximize the EE of the system whilst satisfying certain constraints in terms of maximum transmit power and minimum harvested energy per user. The coupling of the optimization variables, namely, transmit covariance matrices and TS ratios, leads to a EE problem which is non-convex, and hence very difficult to solve directly. Hence, we transform the original maximization problem with multiple constraints into a min-max problem with a single constraint and multiple auxiliary variables. We propose a dual inner/outer layer resource allocation framework to tackle the problem. For the inner-layer, we invoke an extended SWIPT-based BC-multiple access channel (MAC) duality approach and provide two iterative resource allocation schemes under fixed auxiliary variables for solving the dual MAC problem. A sub-gradient searching scheme is then proposed for the outer-layer in order to obtain the optimal auxiliary variables. Numerical results confirm the effectiveness of the proposed algorithms and illustrate that significant performance gain in terms of EE can be achieved by adopting the proposed extended BC-MAC duality-based algorithm.

Index Terms

Simultaneous wireless information and power transfer (SWIPT), energy efficiency (EE), multiple-input multiple-output (MIMO), broadcast channel (BC), dirty paper coding (DPC).

I. Introduction

Energy harvesting (EH) is considered a prominent solution for prolonging the lifetime of power-constrained wireless devices [1]. In addition, EH enhances sustainability by empowering wireless nodes to collect energy from the surrounding environment. In addition to well-recognized renewable energy sources such as biomass, wind, and solar, wireless power transfer (WPT) has emerged as a new enabler for EH. With WPT, the transmitter can transfer energy to the receivers via ambient radio frequency (RF) electromagnetic waves [2].

The integration of RF-based EH capability in communication systems opens up the possibility for simultaneous wireless information and power transfer (SWIPT). This topic has attracted great attention in both academia and industry recently. The authors in [3] investigated practical beamforming techniques in a multiple-input multiple-output (MIMO) SWIPT system, where two practical receiver approaches, namely time-switching (TS) and power-splitting (PS), were discussed. The work in [4], studied the robust beamforming problem for a multi-antenna SWIPT wireless broadcasting system considering imperfect channel state information (CSI) at the transmitter. In [5], a dynamic switching strategy was proposed for a point-to-point SWIPT link in order to exploit the trade-off between information decoding (ID) and EH based on the TS technique. The authors in [6] evaluated the optimal PS ratio in an amplify-and-forward (AF) wireless cooperative network. The work in [7] investigated the optimal PS strategy which achieves the rate-energy region in a single-input single-output (SISO) SWIPT system. This result was then extended in [7] to the case with multiple antennas at the receiver. Further, SWIPT was studied in the context of multi-user orthogonal frequency division multiplexing access (OFDMA) in [8], multi-user multiple-input single-output (MISO) in [9], physical (PHY)-layer security for multicasting in [10], and cognitive radio (CR) in [11].

Most research works on SWIPT systems aim to maximize the rate or the harvested energy, or otherwise achieve a certain rate-energy balance. Nevertheless, the stand-alone maximization of the system throughput would inherently constitute to the highest network power consumption [12]. This trend goes against global commitments for tackling the so-called capacity crunch in a sustainable and economically viable manner. On the other hand, the sole goal of maximizing the harvested energy may degrade the delivered information, and in turn quality of service (QoS). An alternative strategy is therefore to consider energy-efficiency (EE), defined as the number of delivered bits per unit energy [13]. Thanks to the rapid resurgence in green radio (GR) research, EE is nowadays considered a fundamental performance metric in the design and deployment of wireless

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networks [14]. In addition to the many works on the EE optimization problem for conventional communication setups [15]–[17], the maximization of EE has been recently considered in the context of SWIPT systems [18]–[20]. The work in [18] provided a resource allocation algorithm for OFDMA-based SWIPT systems considering a PS-based receiver with continuous sets of PS ratios. In [19], the EE optimization problem in the downlink of a multi-user MISO SWIPT cellular setup was studied, where zero-forcing (ZF) beamforming was employed at the base station (BS). In [20], the authors investigated the EE in the downlink of multi-cell multi-user SWIPT cellular networks. In particular, the authors developed a coordinated precoding scheme using a PS-based minimum mean square error (MMSE) receiver.

A. Main Contributions

In contrast to previous literature on EE for SWIPT, such as multi-carrier OFDMA systems [18], [21], [22], MISO systems based on a fixed precoder such as ZF [19] or MMSE [20], in this paper, we address the EE optimization problem for SWIPT-based MIMO-broadcast channel (BC) where TS technique is employed at each receiver. Particularly, transmit covariance matrices and TS ratios allocation policies are jointly considered towards optimizing the system EE. In addition, apart from the conventional maximum power condition, per-user minimum harvested energy constraints are explicitly included in the EE maximization problem.

Intuitively, by coupling the optimization variables in terms of transmit covariance matrices and TS ratios, the EE maximization problem under consideration becomes non-convex. Hence, it is very difficult to obtain the system EE solution using direct methods, in particular, considering that the algorithm should be feasible in practice. Hence, to tackle this problem, we transform the original optimization problem with multiple constraints into a min-max problem with a single constraint and multiple auxiliary variables. In order to tackle the transformed problem, we propose a dual inner/outer layer resource allocation framework. By invoking the conventional BC-multiple access channel (MAC) duality principle [23], we formulate a dual SWIPT-based MIMO-MAC EE optimization problem with fixed auxiliary variables and accordingly provide an iterative resource allocation algorithm based on the Dinkelbach method [24] for obtaining the solution. Furthermore, in order to reduce the computational complexity, we prove that there exists a quasi-concave relationship between the EE and the transmit power in the dual MAC EE optimization problem. By exploiting the quasi-concavity of the EE in the transmit power, we develop a low-complexity resource allocation scheme based on the particular EE-power region. A sub-gradient searching scheme is then proposed in order to reach the optimal auxiliary variables in the outer-layer. Simulation results confirm the validity of the theoretical findings.

B. Paper Organization

The remainder of this paper is organized as follows. The system model and problem formulation is given in Section II. In Section III, the equivalent EE optimization problem based on the extended BC-MAC duality principle is introduced. In Section IV, an iterative resource allocation scheme based on Dinkelbach method is proposed. In Section V, an alternative low-complexity solution based on the quasi-concavity property of EE-power is proposed. A complete solution to the SWIPT-based EE maximization problem is presented in Section VI. Simulation results are provided in Section VII. Finally, conclusions are drawn in Section VIII.

Notation: bold upper and lower case letters are used to denote matrices and vectors; $(\cdot)^{-1}$ stands for the matrix inverse, $(\cdot)^{T}$ is the matrix transpose; $(\cdot)^{H}$ corresponds to the matrix conjugate transpose; $\mathbf{I}_{N_t \times N_t}$ is an $N_t \times N_t$ identity matrix; Tr$(\cdot)$ denotes the trace of a matrix; $[x]^{+}$ represents $\max(x, 0)$; $(\cdot)^{b}$ and $(\cdot)^{m}$ correspond to BC and MAC parameters, respectively.

II. Preliminaries

In this section, we introduce the system model of a MIMO-BC with TS-based SWIPT and mathematically formulate the EE optimization problem.

A. System Model

The system consists of a BS with N_T transmit antennas and K users $k \in \{1, 2, \ldots, K\}$ each with N_R receive antennas. We denote the channel matrix from the BS to the k^{th} user as $\mathbf{H}_k \in \ C^{N_r \times N_t}$. Channel state information (CSI) is assumed to be perfectly known at the corresponding transmitter and receivers. Note that the CSI at the receivers can be obtained from the channel estimation of the downlink pilots. CSI at the transmitter can be acquired via uplink channel estimation in time division duplex (TDD) systems. Different from the conventional MIMO downlink system [25], each transmission block in the SWIPT-based MIMO-BC system is divided into two orthogonal time slots, one for ID and the other for EH, per illustrated in in Fig. 1. In particular, the TS-based receiver periodically switches its operations between ID and EH. It is assumed that time synchronization has been perfectly built between the transmitter and the receiver. Hence, the received signal from the BS to the k^{th} user before TS can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k, \tag{1}$$

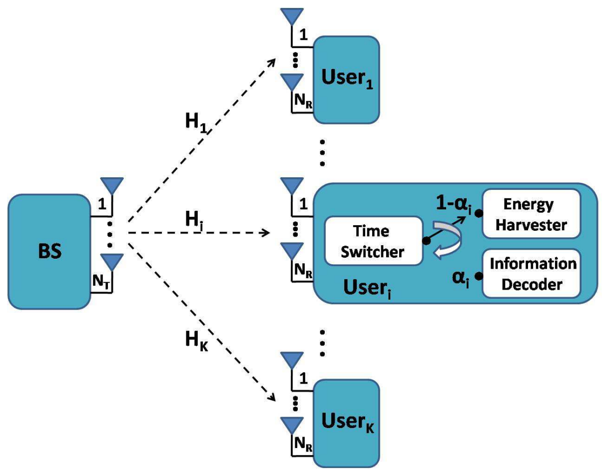

Fig. 1. Schematic diagram of the downlink of a SWIPT-based MIMO-BC system with TS receivers.

where $\mathbf{n}_k \in C^{N_r \times 1}$ is the independent zero mean Gaussian noise with each entry $\mathcal{CN}(0, \sigma^2)$, $\mathbf{x}$ is the transmitted signal on the downlink. In addition, $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_K$ where $\mathbf{x}_k \in C^{N_t \times 1}$, is the signal transmitted to the k^{th} user [26].

It is important to note that it is unnecessary to convert the RF band signal to the baseband for the purpose of collecting the carried energy. However, because of the law of conservation of energy, we could make a reasonable assumption that the total harvested RF-band power (from all receiving antennas) is proportional to that of the received baseband signal. In addition, we note that the structure of an EH-based receiver depends on its specific implementation in practical wireless communication systems. For instance, electromagnetic induction and electromagnetic radiation are capable of transferring wireless energy [27]. On the other hand, the receivers' hardware circuitries as well as the corresponding EH efficiency could be significantly different. Apart from that, due to the practical hardware limitations, the decoding signal cannot be used for collecting energy directly [27]. Consequently, to avoid the impact from the specific hardware implementation details on the resource allocation algorithm design, we do not assume a particular type of EH receiver. In this paper, receivers consisting of a harvesting energy unit and a conventional signal processing unit for concurrent EH and ID is under consideration. Let α_k with $0 \leq \alpha_k \leq 1$ denote the percentage of transmission time allocated to the ID time slot for user k. Thus, $1 - \alpha_k$ corresponds to the percentage of transmission time allocated to the EH time slot for user k. Hence, the harvested energy at the receiver of user k can be written as

$$E_k = (1 - \alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k \mathbf{Q}), \tag{2}$$

where η_k is a constant that accounts for the loss in the energy transducer for converting the harvested energy to electrical energy to be stored, $\mathbf{G}_k = \mathbf{H}_k^H \mathbf{H}_k$ is the channel covariance matrix, $\mathbf{Q}$ denotes the total transmit covariance matrix at the BS, $\mathbf{Q} = \sum_{k=1}^{K} \mathbf{Q}_k^b$, $\mathbf{Q}_k^b = \mathbb{E}(\mathbf{x}_k \mathbf{x}_k^H)$ is the corresponding transmit covariance matrix, $\mathbf{Q}_k^b \succeq 0$ i.e., $\mathbf{Q}_k^b$ is a positive semidefinite matrix. On the other hand, the total capacity of the MIMO-BC SWIPT system can be expressed as follows

$$C_{BC} = \sum_{k=1}^{K} \alpha_k R_k^b, \tag{3}$$

where R_k^b denotes the rate achieved by the user k in the downlink. Further, note that dirty paper coding (DPC) is the capacity achieving scheme for Gaussian MIMO-BC [28]. With DPC, the information for different users is encoded in a sequential fashion. It should be noted that the transmit covariance matrices remain the same during each transmission block. This implies that the same transmit covariance matrices are shared in ID and EH models. Without loss of generality, with an encoding order $(1, \cdots, K)$, i.e., the codeword of user 1 is encoded first, the data rate R_k^b for the k^{th} user can be written as [23], [29]

$$R_k^b = W \log \frac{|\mathbf{I}_{N_r \times N_r} + \frac{1}{\sigma^2} \mathbf{H}_k (\sum_{i=k}^{K} \mathbf{Q}_i^b) \mathbf{H}_k^H|}{|\mathbf{I}_{N_r \times N_r} + \frac{1}{\sigma^2} \mathbf{H}_k (\sum_{i=k+1}^{K} \mathbf{Q}_i^b) \mathbf{H}_k^H|}. \tag{4}$$

Defining a quantitative power model is very challenging given one needs to consider the particular deployment scenario and components configurations [30], [31]. The following linear power model is widely shown to be a reasonable representative for radio access networks [32]

$$P = \zeta P_T + P_C, \tag{5}$$

where ζ, P_T and P_C are respectively used to denote the reciprocal of the power amplifier drain efficiency, transmission power, and circuit power consumption. However, for the SWIPT-based MIMO-BC system considered in this work, the power consumption model should be extended considering EH devices. In general, small amounts of energy is consumed by the RF EH devices. On the other hand, the system power consumption is intuitively compensated by the harvested energy. It may then be apparent that enabling EH can improve the EE of a wireless communication system. Thus, as in [22], [33], we take the harvested power into consideration in the formulation of the system power consumption model (and hence the EE formulation for the SWIPT-based MIMO-BC system).

Specifically, the total system power consumption is expressed as follows

$$P = \zeta P_T + P_C - \sum_{k=1}^{K} E_k, \tag{6}$$

where $\sum_{k=1}^{K} E_k$ represents the harvested power at all the receivers Note that the minus sign denotes that a portion of the power radiated in the RF from the transmitter can be harvested by the K receivers. Recall that P_C is the total circuit power required for supporting reliable communication in our SWIPT-based MIMO-BC

$$P_C = P_{ant}^{BS} N_T + P_{sta} + K P_C^R, \tag{7}$$

where $P_{ant}^{BS} N_T$ denotes the power consumption proportional to the number of transmit antennas, P_{sta} is the constant signal processing circuit power consumption in the transmitters (due to filters, frequency synthesizer, etc., independent of the power radiated by the transmitter), and P_C^R denotes the total circuit power consumption in the K receivers.

B. Problem Formulation

In this work, we employ DPC which achieves the sum rate capacity for SWIPT-based MIMO-BC. It should be noted that due to certain practical constraints, such as pilot overhead, coding and modulation, demodulation and decoding, etc., there exists a performance gap between the channel capacity and the achievable rate [34]. Nevertheless, the sum rate capacity obtained by DPC scheme under perfect CSI represents the information-theoretic upper bound for MIMO-BC, which helps unveil important insights into the problem.

The EE for SWIPT-based MIMO-BC can be defined as the total number of delivered bits per unit energy. The energy consumption includes transmission energy consumption, circuit energy consumption, as well as harvested energy. Hence, we define EE in a SWIPT-based MIMO-BC as

$$\lambda_{EE} \triangleq \frac{C_{BC}}{P} = \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^{K} E_k}, \tag{8}$$

where C_{BC} is the total capacity achieved by all users and $P_T = \sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^b)$ is the total transmission power.

Given the expression of the system sum rate and power consumption, we can proceed with the problem formulation. The objective of this paper is to maximize the EE in SWIPT-based MIMO-BC whilst meeting two constraints in terms of transmission power and harvested energy. By invoking the linear power model in (6), the optimization problem can be formulated as

$$\max_{\mathbf{Q}_k^b \succeq 0, \alpha_k} \quad \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^{K} (1-\alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k \mathbf{Q})}, \tag{9}$$

$$\text{s.t.} \quad (1-\alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k \mathbf{Q}) \geq E_{k,min}, \ \forall\, k \in \mathcal{K}, \tag{10}$$

$$\sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^b) \leq P_{max}, \tag{11}$$

$$0 \leq \alpha_k \leq 1, \ \forall\, k \in \mathcal{K}, \tag{12}$$

where P_{max} and $E_{k,min}$ are the maximum total transmit power constraint at the BS and the minimum harvested energy constraint for user $k \in (1, 2, \cdots, K)$, respectively. Note that (12) corresponds to the inherent constraints in terms of TS ratios. It is easy to see that the coupling of optimization variables leads to the problem (9)-(12) being non-convex and challenging to solve directly. The complexity is considered the main drawback for PHY algorithm design. Therefore, in the following sections, we develop resource allocation schemes for SWIPT-based MIMO-BC to solve the above optimization problem. In particular, to overcome the difficulty, we first transform this multi-constrained EE maximization problem into its equivalent problem that has a single constraint with multiple auxiliary variables. We then, for the first time, exploit the duality between a SWIPT-based MIMO-BC and a dual MIMO-MAC in the case where the multiple auxiliary variables are fixed. For the dual-MAC problem, we accordingly propose two iterative resource allocation algorithms based on Dinkelbach theory [24] and EE-power quasi-concavity property.

III. Equivalence and Extended BC-MAC Duality

Evidently, the SWIPT-based MIMO-BC EE maximization problem under the maximum total transmit power constraint and multiple minimum harvested energy constraints is a non-convex optimization problem and is difficult to solve directly. However, it is shown in [23] that under a single sum power constraint, the weighted sum rate maximization problem for the MIMO-BC can be transformed to its dual MIMO-MAC problem, which is convex and can be solved in an efficient manner. Furthermore, adding an interference constraint to form a cognitive radio MIMO-BC scenario, the authors in [35] proved this BC-MAC duality still holds for a weighted sum rate maximization problem. Nevertheless, in our SWIPT-based MIMO-BC setting, the EE optimization problem in (9)-(12) has not only a sum power constraint but also multiple minimum harvested energy constraints. The imposed multiple constraints further complicate the formulation of an efficient solvable dual problem. In order to overcome the aforementioned challenges, we first transform this multi-constrained EE maximization problem in (9)-(12) into its equivalent problem that has a single constraint with multiple auxiliary variables. Thus, we develop a duality between a SWIPT-based MIMO-BC and a SWIPT-based dual MIMO-MAC in the case where the multiple auxiliary variables are fixed. In the following, we present an equivalent form of the problem in (9)-(12).

Proposition 1: *The problem in (9)-(12), and hence its solution, is equivalent to*

$$\min_{\chi\geq 0,\mu_k\geq 0}\ \max_{\boldsymbol{Q}_k^b\succeq 0, 0\leq\alpha_k\leq 1}\ \frac{\sum_{k=1}^K \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^K (1-\alpha_k)\eta_k tr(\boldsymbol{G}_k\boldsymbol{Q})} \tag{13}$$

$$s.t.\ \ \chi\left(\sum_{k=1}^K tr(\boldsymbol{Q}_k^b) - P_{max}\right) + \sum_{k=1}^K \mu_k\left(E_{k,min} - (1-\alpha_k)\eta_k tr(\boldsymbol{G}_k\boldsymbol{Q})\right) \leq 0, \tag{14}$$

where χ and μ_k are the auxiliary dual variables for the maximum power constraint and the k^{th} minimum harvested energy constraint, respectively.

Proof: See Appendix A.

However, it is still very difficult to directly find an efficiently solvable dual problem for (13)-(14). Thus, in the following, we first investigate the EE maximization problem considering fixed auxiliary dual variables χ and μ_k. The problem in (13)-(14) can hence be reduced to

$$\max_{\mathbf{Q}_k^b\succeq 0, 0\leq\alpha_k\leq 1}\ \frac{\sum_{k=1}^K \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^K\sum_{k=1}^K (1-\alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k\mathbf{Q})} \tag{15}$$

$$\text{s.t.}\ \ \chi\left(\sum_{k=1}^K \mathrm{tr}(\mathbf{Q}_k^b) - P_{\max}\right) + \sum_{k=1}^K \mu_k(E_{k,min} - (1-\alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k\mathbf{Q})) \leq 0. \tag{16}$$

The solution of the above problem is unfortunately non-trivial given the objective function is non-concave even under fixed auxiliary dual variables χ and μ_k. Thus, we exploit an extended SWIPT-based BC-MAC duality principle based on results from [23] and [35]. Consequently, the weighted sum rate maximization problem in SWIPT-based MIMO-BC under constraints in (14) can be formulated as

$$\max_{\mathbf{Q}_k^b\succeq 0, 0\leq\alpha_k\leq 1}\ \sum_{k=1}^K \alpha_k R_k^b, \tag{17}$$

$$\text{s.t.}\ \ \chi\sum_{k=1}^K \mathrm{tr}(\mathbf{Q}_k^b) - \sum_{k=1}^K \mu_k(1-\alpha_k)\eta_k \mathrm{tr}(\mathbf{G}_k\mathbf{Q}) \leq P_{all}, \tag{18}$$

where $P_{all} := \chi P_{\max} - \sum_{k=1}^K \mu_k E_{k,min}$. Since χ and μ_k are fixed, P_{all} is a constant (17)-(18). Hence, by extending the general BC-MAC duality principle from [23] to our SWIPT-based MIMO-BC scenario, we have the following SWIPT-based dual MAC problem corresponding to the original SWIPT-based BC problem in (17)-(18).

Proposition 2: *The SWIPT-based dual MAC problem of (17)-(18) is given by*

$$\max_{\boldsymbol{Q}_k^m\succeq 0, 0\leq\alpha_k\leq 1}\ \sum_{k=1}^K \alpha_k R_k^m \tag{19}$$

$$s.t.\ \ \sum_{k=1}^K tr(\boldsymbol{Q}_k^m) \leq P_{all}, \tag{20}$$

where $\boldsymbol{Q}_k^m$ is the transmit signal covariance matrix of the k^{th} user, and R_k^m is the rate achieved by the k^{th} user of the dual MAC defined as

$$R_k^m = W\log\frac{|\boldsymbol{N} + \boldsymbol{H}_k^H(\sum_1^i \boldsymbol{Q}_i^m)\boldsymbol{H}_k|}{|\boldsymbol{N} + \boldsymbol{H}_k^H(\sum_1^{i-1} \boldsymbol{Q}_i^m)\boldsymbol{H}_k|}. \tag{21}$$

with the noise covariance at the BS denoted with $\mathbf{N}=\chi\boldsymbol{I}-\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k[tr(\boldsymbol{G}_k\boldsymbol{Q})]$).

Proof: See Appendix B.

Proposition 2 indicates that the capacity region of a SWIPT-based MIMO-BC with power constraint P_{all} is equal to the union of capacity regions of the SWIPT-based dual-MAC with power constraints such that $\sum_{k=1}^{K}\mathrm{tr}(\mathbf{Q}_k^m)=P_{all}$. However, *Proposition 2* describes the rate region for SWIPT-based MIMO-BC system and its duality relationship with SWIPT-based MIMO-MAC, meaning the EE aspect is still an open question. Hence, in order to tackle the problem in (15)-(16), we develop the following proposition (EE aspect) based on the results in *Proposition 2*.

Proposition 3: *The solution of the SWIPT-based dual MAC EE maximization problem, namely,*

$$\max_{\boldsymbol{Q}_k^m\succeq 0, 0\leq\alpha_k\leq 1}\ \frac{\sum_{k=1}^{K}\alpha_k R_k^m}{\zeta tr(\boldsymbol{Q}_k^m)+P_C} \tag{22}$$

$$s.t.\ \ \sum_{k=1}^{K} tr(\boldsymbol{Q}_k^m)\leq P_{all}, \tag{23}$$

is an upper-bound of the solution to the problem in (15)-(16).

Proof: See Appendix C.

Consequently, instead of directly tackling the EE maximization problem in (15)-(16), in this work, we have provided a dual-MAC upper bound solution of (22)-(23). It should be noted that due to the additional minimum energy harvesting constraints and the TS ratio variables α_k, the optimization problem is a non-linear fractional programming and hence the problem in (22)-(23) cannot be solved via our previous duality result provided in [17], in which only a single sum power constraint was considered. Besides, the TS ratio α_k in the SWIPT system plays a very important role in our optimization problem, hence further increases the complexity for obtaining a solution. For the optimization problems of such nature, it is generally helpful to relate it to a concave program by separating numerator and denominator with the help of parameter β, this is what is known as the Dinkelbach method [24]. In the following sections, we propose an iterative resource allocation algorithm based on the Dinkelbach method to obtain the upper-bound solution to the problem in (15)-(16).

IV. Iterative Resource Allocation Scheme based on Dinkelbach Method

Recall that the optimization problem in (22)-(23) belongs to a family of non-linear fractional programming problems which are non-convex and difficult to solve directly. Nevertheless, by invoking the theory of non-linear fractional programming in [24], we can use the Dinkelbach method to solve this non-convex non-linear fractional programming problem. Specifically, we transform the fractional-form objective function into a numerator-denominator subtractive form as discussed in the following proposition.

Proposition 4: *The maximum EE* β^* *can be achieved if and only if*

$$\max_{\mathbf{Q},\boldsymbol{\alpha}}\ U_R(\mathbf{Q},\boldsymbol{\alpha})-\beta^* U_T(\mathbf{Q},\boldsymbol{\alpha})=U_R(\mathbf{Q}^*,\boldsymbol{\alpha}^*)-\beta^* U_T(\mathbf{Q}^*,\boldsymbol{\alpha}^*)=0 \tag{24}$$

for $U_R(\mathbf{Q},\boldsymbol{\alpha})\geq 0$ and $U_T(\mathbf{Q},\boldsymbol{\alpha})\geq 0$, where

$$U_R(\mathbf{Q},\boldsymbol{\alpha})=\sum_{k=1}^{K}\alpha_k R_k^m, \tag{25}$$

$$U_T(\mathbf{Q},\boldsymbol{\alpha})=\zeta\sum_{k=1}^{K}\mathrm{tr}(\mathbf{Q}_k^m)+P_C, \tag{26}$$

$$\text{and}\ \ \beta^*=\frac{U_R(\mathbf{Q}^*,\boldsymbol{\alpha}^*)}{U_T(\mathbf{Q}^*,\boldsymbol{\alpha}^*)}. \tag{27}$$

Proof: Please refer to [24] for a proof of *Proposition 4*.

Proposition 4 offers a sufficient and necessary condition for designing the optimal resource allocation strategy. Particularly, we can find an equivalent optimization problem with an objective function in subtractive form, e.g. $U_R(\mathbf{Q},\boldsymbol{\alpha})-\beta^* U_T(\mathbf{Q},\boldsymbol{\alpha})$ in the considered case, corresponding to the original optimization problem with an objective function in fractional form, such that both problems share the same optimal solution (same optimal resource allocation strategy). In addition, the optimality guarantees the equality in (24), and thus we could apply this equality condition to verify the optimality of the solution. Therefore, we develop a resource allocation strategy for the equivalent objective function (subtractive form) whilst satisfying the condition stated in *Proposition 4*.

Based on Dinkelbach method [24], here, we propose an iterative algorithm for solving (22)-(23) with an equivalent objective function such that the obtained solution satisfies the conditions stated in *Proposition 4*. The proposed algorithm is summarized in Table I.

TABLE I
PROPOSED ITERATIVE RESOURCE ALLOCATION ALGORITHM BASED ON DINKELBACH METHOD

1) Initialize $\beta = 0$, and δ as the maximum tolerance;
2) **REPEAT**
3) For a given β, obtain an intermediate resource allocation policy $\{\mathbf{Q}, \boldsymbol{\alpha}\}$ by solving the problem in (28)-(29);
4) **IF** $U_R(\mathbf{Q}, \boldsymbol{\alpha}) - \beta U_T(\mathbf{Q}, \boldsymbol{\alpha}) \leq \delta$
5) Convergence = **TRUE**;
6) **RETURN** $\{\mathbf{Q}^*, \boldsymbol{\alpha}^*\}$ = $\{\mathbf{Q}, \boldsymbol{\alpha}\}$ and $\beta^* = \frac{U_R(\mathbf{Q},\boldsymbol{\alpha})}{U_T(\mathbf{Q},\boldsymbol{\alpha})}$;
7) **ELSE**
8) Set $\beta = \frac{U_R(\mathbf{Q},\boldsymbol{\alpha})}{U_T(\mathbf{Q},\boldsymbol{\alpha})}$ and $n = n + 1$, Convergence = **FALSE**;
9) **END IF**
10) **UNTIL** Convergence = **TRUE**.

It can be observed from Table I that the key step for the proposed iterative algorithm concerns the solution to the following optimization problem for a given parameter β in each iteration (i.e., step 3),

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \ \sum_{k=1}^{K} \alpha_k R_k^m - \beta \left(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C \right), \tag{28}$$

$$\text{s.t.} \ \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \leq P_{all}. \tag{29}$$

To solve this problem, we define $f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K) = \Delta_k \log |\mathbf{N} + \sum_{k=1}^K \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k|$, where $\Delta_k = \alpha_k - \alpha_{k+1}$, and thus the optimization problem in (28)-(29) can be reformulated as

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \ f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K) - \beta \left(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C \right), \tag{30}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \leq P_{all}. \tag{31}$$

The corresponding Lagrangian function can be expressed as

$$\begin{aligned} L(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K, \tau) &:= f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K), \\ &-\beta \left(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C \right) - \tau \left[\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) - P_{all} \right], \end{aligned} \tag{32}$$

where $\tau \geq 0$ is the Lagrangian multipliers associated with the maximum power constraint. The dual objective function of (28) is written as

$$g(\tau) = \max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \ L(\mathbf{Q}_1^m, \cdots \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K, \tau), \tag{33}$$

and the dual problem is given by

$$\min_{\tau} g(\tau) \quad s.t. \quad \tau \geq 0. \tag{34}$$

In this work, an iterative approach is used here in order to achieve the optimum $\mathbf{Q}_k^m$ and α_k for the dual MIMO-MAC problem. In particular, we update $\mathbf{Q}_k^m$ through the gradient of the Lagrangian function (32) with respect to $\mathbf{Q}_k^m$ and α_k as follows

$$\nabla_{\mathbf{Q}_k^m} L := \frac{\partial f[\mathbf{Q}_1^m(n), \cdots, \mathbf{Q}_{k-1}^m(n), \mathbf{Q}_k^m(n-1), \cdots, \mathbf{Q}_K^m(n-1)]}{\partial \mathbf{Q}_k^m(n-1)} - \beta \zeta \tau \mathbf{I}_{N_r \times N_r}, \tag{35}$$

$$\nabla_{\alpha_k} L := \frac{\partial f[\alpha_1(n), \cdots, \alpha_{k-1}(n), \alpha_k(n-1), \cdots, \alpha_K(n-1)]}{\partial \alpha_k(n-1)}, \tag{36}$$

$$\mathbf{Q}_k^m(n) = [\mathbf{Q}_k^m(n-1) + t \nabla_{\mathbf{Q}_k^m} L]^+, \tag{37}$$

$$\alpha_k(n) = \alpha_k(n-1) + t \nabla_{\alpha_k} L, \tag{38}$$

where t represents the step size, and the notation $[\mathbf{A}]^+$ is defined as $[\mathbf{A}]^+ := \sum_i [q_i]^+ \mathbf{v}_i \mathbf{v}_i^H$, with q_i and $\mathbf{v}_i$ denote the i^{th} eigenvalue and the corresponding eigenvector of $\mathbf{A}$ respectively. Therefore, we can compute the gradient in (35) and (36) as

TABLE II
BISECTION-BASED RESOURCE ALLOCATION ALGORITHM

1) Initialize τ_{min} and τ_{max};
2) **REPEAT**
3) $\tau = (\tau_{\text{min}} + \tau_{\text{max}})/2$;
4) **REPEAT** Initialize $\mathbf{Q}_1^m(0), \cdots, \mathbf{Q}_K^m(0)$, $\alpha_1(0), \cdots, \alpha_K(0)$, $n = 1$;
5) **FOR** $k = 1, \cdots, K$
6) $\mathbf{Q}_k^m(n) = [\mathbf{Q}_k^m(n-1) + t\nabla_{\mathbf{Q}_k^m} L]^+$;
7) $\alpha_k(n) = \alpha_k(n-1) + t\nabla_{\alpha_k} L$;
8) **END FOR**
9) $n = n + 1$;
10) **UNTIL** $\mathbf{Q}_k^m$ and α_K for $k = 1, \cdots, K$ converge, i.e., $\|\nabla_{\mathbf{Q}_k^m} L\|^2 \leq \epsilon$, $\|\nabla_{\alpha_k} L\|^2 \leq \epsilon$ for a small ϵ;
11) if $\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^m) \geq P_{all}$, $\tau_{\text{min}} = \tau$, elseif $\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^m) \leq P_{all}$, $\tau_{\text{max}} = \tau$;
12) **UNTIL** $\lvert\tau_{\text{min}} - \tau_{\text{max}}\rvert \leq \varepsilon$.

follows

$$\frac{\partial f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m)}{\partial \mathbf{Q}_k^m} = \mathbf{H}_k \left(\mathbf{I}_{N_t \times N_t} + \frac{1}{\sigma^2} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{Q}^m{}_k \mathbf{H}_k \right)^{-1} \mathbf{H}_k^H. \tag{39}$$

$$\frac{\partial f(\alpha_1, \cdots, \alpha_K)}{\partial \alpha_k} = \sum_{i=1}^{K} \Delta_i \text{tr} \left[\mu_i \eta_i \mathbf{G}_i \left(\sum_{k=1}^{i} \mathbf{N} + \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k \right)^{-1} \right] + R_k^m. \tag{40}$$

After we obtain the optimum $\mathbf{Q}_k^m$ and α_k, our next task is to find out the optimal τ. Given that the Lagrangian function $g(\tau)$ is a convex function with respect to τ, we can achieve the optimal τ through a one-dimensional searching approach. Nevertheless, it is not guaranteed that $g(\tau)$ is differentiable, and thus the gradient approach may not available in this case. On the other hand, the well-known sub-gradient approach can be applied here to search the optimal solution where τ is updated in accordance with the sub-gradient direction as the following Lemma.

Lemma 1. *$P_{all} - \sum_{k=1}^K tr(\boldsymbol{Q}_k^m)$ is the sub-gradient of the dual objective function $g(\tau)$, where $\boldsymbol{Q}_k^m, k = 1, 2, \cdots, K$ are the corresponding optimal covariance matrices under fixed τ.*

Proof: See Appendix D.

Upon convergence of the transmit covariance matrices $\mathbf{Q}_k^m, k = 1, 2, \cdots, K$ and the TS ratios $\alpha_k, k = 1, 2, \cdots, K$, the current consumption power is saved in order to compare with P_{all}. In particular, as stated in *Lemma 1*, the value of τ should be increased if $\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^m) \geq P_{all}$, and decrease otherwise. This procedure is continued until convergence, i.e., $\lvert\tau_{\text{min}} - \tau_{\text{max}}\rvert \leq \varepsilon$.

We can now present the algorithm to solve the optimization problem for a given parameter β in each iteration, namely the bisection-based resource allocation algorithm, as in Table II.

A. Special Case with Equal TS Ratios

In the previous section, we investigated the case in which different SWIPT-equipments (receivers) are employed by the users, and hence the optimal TS ratio factors for users was assumed to vary (depending on their channel gain and the minimum EH constraints). However, in a practical communications system, it is very likely that all served users are equipped with the same SWIPT-equipments, resulting in $\alpha = \alpha_1 = \alpha_2 \cdots = \alpha_K$. As a result, here, we develop an efficient solution for solving the equal TS ratio case.

Motivated by the iterative approach proposed in [36], we here separate the process of determining the TS ratio α and the covariance matrix $\mathbf{Q}$ as follows

$$\underbrace{\boldsymbol{\alpha}[0] \to \mathbf{Q}[0]}_{\text{Initialization}} \to \cdots \underbrace{\boldsymbol{\alpha}[t] \to \mathbf{Q}[t]}_{\text{Iteration t}} \to \underbrace{\boldsymbol{\alpha}^{opt} \to \mathbf{Q}^{opt}}_{\text{Optimal Solution}}. \tag{41}$$

In particular, a bi-section approach is proposed on the basis of the following proposition.

Proposition 5: *For any given transmit covariance matrices $\boldsymbol{Q}_k^m, k = 1, 2, \cdots, K$, that satisfies the constraints in (29), the following optimization problem,*

$$\max_{\alpha}\ \vartheta(\alpha) = \max_{\alpha}\ \sum_{k=1}^{K} \alpha R_k - \beta \left(\zeta \sum_{k=1}^{K} tr(\boldsymbol{Q}_k^m) + P_C \right) \tag{42}$$

$$s.t.\ \ 0 \leq \alpha \leq 1, \tag{43}$$

TABLE III
BISECTION-BASED RESOURCE ALLOCATION ALGORITHM FOR THE EQUAL TS RATIO CASE.

1) Initialize $\alpha_{\text{min}} = 0$ and $\alpha_{\text{max}} = 1$;
2) **REPEAT**
3) $\alpha = (\alpha_{\text{min}} + \alpha_{\text{max}})/2$;
4) **REPEAT**, Initialize $\mathbf{Q}_1(0), \cdots, \mathbf{Q}_K(0)$, $n = 1$;
5) **FOR** $k = 1, \cdots, K$
6) $\mathbf{Q}_k(n) = [\mathbf{Q}_k(n-1) + t\nabla_{\mathbf{Q}_k} L]^+$;
7) **END FOR**
8) $n = n + 1$;
9) **UNTIL** $\mathbf{Q}_k$ for $k = 1, \cdots, K$ converge;
10) if $\nabla_\alpha \vartheta \geq 0$, $\alpha_{\text{min}} = \alpha$, elseif $\nabla_\alpha \vartheta \leq 0$, $\alpha_{\text{max}} = \alpha$;
11) **UNTIL** $\lvert\alpha_{\text{min}} - \alpha_{\text{max}}\rvert \leq \varepsilon$.

is concave in α.
Proof: See Appendix E.

Consequently, *Proposition 5* guarantees the existence and uniqueness of the global maximum solution. Furthermore, $\vartheta(\alpha)$ either strictly decreases or first increases and then strictly decreases with α. Therefore, problem (28)-(29) can be decomposed into two layers and solved iteratively through the bi-section processes, per detailed in Table III.

V. ALTERNATIVE SOLUTION BASED ON QUASI-CONCAVITY PROPERTY

The proposed iterative resource allocation scheme for the problem in (22)-(23) is based on the Dinkelbach method, and hence the convergence speed for β may be slow for some special cases, i.e., when large number of users exists in the network. To facilitate practical implementation of the optimal resource efficient design, we will exploit and prove the quasi-concave relation between the maximum EE $\lambda_{EE}^*(P_T^m)$ and transmit power $P_T^m = \sum_{k=1}^K \text{tr}(\mathbf{Q}_k^m)$. In particular, we first demonstrate the quasi-concavity of EE in transmit power, and then develop a dual-layer resource allocation scheme based on the EE-power relationship.

A. Fundamentals of EE-Power Relationship

We proceed by providing a fundamental study of the EE-power relationship.

Proposition 6: *With transmit covariance matrices* $\boldsymbol{Q}_k^m, k = 1, 2, \cdots, K$ *and TS ratios* $\alpha_k, k = 1, 2, \cdots, K$*, that satisfies the constraints in (23), i.e.,* $P_T^m \leq P_{max}$*, the maximum EE,* $\lambda_{EE}^* = \max\limits_{\boldsymbol{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \lambda_{EE}(P_T^m)$*, is strictly quasi-concave in* P_T^m*.*
Proof: See Appendix F.

Proposition 7: *For any given transmission power in the region* $[P_{min}, P_{max}]$*, the maximum EE,* $\lambda_{EE}^*(P_T^m)$*, is*
(i) strictly decreasing with P_T^m *and is maximized at* $P_T^m = P_{min}$ *if*

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} \leq 0,$$

(ii) strictly increasing with P_T^m *and is maximized at* $P_T^m = P_{max}$ *if*

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} > 0$$

$$\text{and} \quad \left.\frac{d\lambda_{EE}^*(P)}{dP_T^m}\right|_{P_T^m = P_{max}} \geq 0,$$

(iii) first strictly increasing and then strictly decreasing with P_T^m *and is maximized at* $P_T^m = \frac{\bar{\lambda}_{EE}}{C_{BC}(\bar{\lambda}_{EE})} - P_C$ *if*

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP}\right|_{P_T^m = P_{min}} > 0$$

$$\text{and} \quad \left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{max}} \leq 0,$$

(iv) infeasible if

$$P_{min} > P_{max},$$

where $C_{BC}(\bar{\lambda}_{EE})$ represents the capacity under maximum achievable EE.
Proof: See Appendix G.

Note that a quasi-concavity property guarantees the existence of a unique maximum solution, and thus *Proposition 6* ensures the existence and uniqueness of the maximum solution. Moreover, the quasi-concavity further indicates that $\lambda_{EE}(P_T^m)$ either strictly decreases or first increases and then strictly decreases with P_T^m. *Proposition 7* further reveals that there exists a maximum point at a finite power region. Thus, the optimization problem in (22)-(23) can be solved through a dual-layer decomposition method using the processes,
(i) inner-layer: For a fixed transmit power, P_T^m, determines the maximum EE $\lambda_{EE}^*(P_T^m)$, and
(ii) outer-layer: obtains the optimal EE, $\bar{\lambda}_{EE}$, through a gradient-based approach.

Note that the key challenge of the proposed dual-layer decomposition method lies in the inner-layer mechanism, where $\lambda_{EE}^*(P_T^m)$ is to be obtained. This is discussed in detail as analysis proceeds.

B. Resource Allocation for the Dual MAC Problem

Recall that the inner-layer is concerned with finding the maximum EE, $\lambda_{EE}^*(P_T^m)$, based on a given transmission power, i.e., any P_T^m in the power region $[P_{min}, P_{max}]$. Hence, the optimization problem with a given transmission power P_T^m can be expressed as

$$\max_{\mathbf{Q}_k^m \succeq 0, 0\le\alpha_k\le 1} \sum_{k=1}^{K} \alpha_k R_k^m \tag{44}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \le P_T^m. \tag{45}$$

Hence, the inner-layer of the proposed algorithm has been transformed to solve the optimization problem in (44)-(51) based on a given transmission power P_T^m. Apparently, the optimization problem in (44)-(51) is similar to the problem in (30), and hence they share the same solution structure. In other words, the proposed bisection-based resource allocation algorithm in Section IV can be applied here to solve the sum rate maximization problem in (44)-(51), and hence the detailed methodology is omitted here.

We next design a searching algorithm in order to solve the outer-layer of the EE optimization problem in (22)-(23). We first initialize the transmit power as $P_T^m(0)$. Based on the fixed transmit power, we determine the maximum EE $\lambda_{EE}^*(P_T^m)$ using the proposed bisection-based resource allocation algorithm in Section IV. We then develop a searching scheme based on *Proposition 7* to update the transmit power P_T^m as follows

$$P_T^m(n) \;=\; \begin{cases} \frac{P_T^m(n-1)}{\varsigma} & \left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m(n-1)} < 0 \\ \varsigma P_T^m(n-1) & \text{otherwise} \end{cases}, \tag{46}$$

where $\varsigma > 1$ denotes the searching step. In addition, the value of ς should be reduced if the sign of gradient $\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}$ is changed as in

$$\varsigma(n) = \frac{\varsigma(n-1)}{2} \tag{47}$$

and (46) is repeated until convergence, i.e., $|P_T^m(n+1) - P_T^m(n)| \le \epsilon$ or either P_{max} or P_{min} is achieved. In other words, the proposed resource allocation algorithm for the dual MAC problem will converge to the optimal point or the boundary point.

VI. Solution to the SWIPT-based EE Maximization Problem

Here, we provide a complete solution to the EE optimization problem in (13)-(14), namely an extended BC-MAC duality-based EE maximization algorithm. Under fixed χ and $\boldsymbol{\mu}$, the problem can be reformulated as follows

$$x(\chi, \boldsymbol{\mu}) = \max_{\mathbf{Q}_k^b \succeq 0, 0\le\alpha_k\le 1} \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T^m + P_C - \sum_{k=1}^{K}\sum_{k=1}^{K}(1-\alpha_k)\eta_k \mathbb{E}[\text{tr}(\mathbf{G}_k\mathbf{Q})]} \tag{48}$$

$$\text{s.t.} \quad \chi\left(\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) - P_{\text{max}}\right) + \sum_{k=1}^{K} \mu_k(E_{k,min} - (1-\alpha_k)\eta_k \mathbb{E}[\text{tr}(\mathbf{G}_k\mathbf{Q})]) \le 0. \tag{49}$$

In addition, the problem (13)-(14) is equivalent to the following

$$\min_{\chi, \boldsymbol{\mu}} \; x(\chi, \boldsymbol{\mu}) \tag{50}$$

$$\text{s.t.} \quad \chi \ge 0 \text{ and } \mu_k \ge 0, \; \forall k \in \mathcal{K}. \tag{51}$$

By applying the BC-MAC duality in Section III together with the proposed Dinkelbach method-based iterative resource allocation scheme in Section IV, or the alternative solution based on quasi-concavity property in Section V, one can achieve $x(\chi, \boldsymbol{\eta})$. We then apply the BC-MAC covariance mapping approach from [35] to obtain the corresponding BC transmit covariance matrices $\mathbf{Q}_k^b, k = 1, \cdots, K$. Once we have obtained the solution for a given χ and $\boldsymbol{\mu}$, we can use the following lemma to update χ and $\boldsymbol{\mu}$ through a sub-gradient approach.

Lemma 2. *The sub-gradient of $x(\chi, \boldsymbol{\mu})$ is $[P_{max} - \sum_{k=1}^{K} tr(\boldsymbol{Q}_k^b), (1-\alpha_k)\eta_k[tr(\boldsymbol{G}_k\boldsymbol{Q})] - E_{k,min}]$, $k = 1, 2, \cdots, K$, where $\chi \geq 0$, $\boldsymbol{\mu} \geq \boldsymbol{0}$ and $\boldsymbol{Q}_k, \alpha_k, k = 1, 2, \cdots, K$, respectively denote the corresponding optimal transmit covariance matrices and TS ratios for a fixed χ and $\boldsymbol{\mu}$ in (50).*

Proof: The proof of *Lemma 2* is similar to that of *Lemma 1*, and thus is omitted for brevity.

It should be noted that with a constant step size, the sub-gradient approach will converge to a point that is very close to the optimal value [37], i.e.,

$$\lim_{n \to \infty} |\chi^n - \chi^*| < \epsilon, \text{ and } \lim_{n \to \infty} |\mu_k^n - \mu_k^*| < \epsilon, \ k = 1, 2, \cdots, K, \tag{52}$$

where χ^* and μ_k^* are the optimal values, and χ^n and μ_k^n are the values of χ and μ_k at the n^{th} iteration of the sub-gradient approach, respectively. This result indicates that the sub-gradient approach determines an ϵ-suboptimal point in a finite number of iterations.

VII. Simulation Results

In this section, we present simulation results to verify the theoretical findings and analyze the effectiveness of the proposed approaches. In our simulation, the BS employs $N_t = 4$ transmit antennas, each user is equipped with $N_R = 2$ receive antennas, and the total number of users is set to $K = 4$. The path-loss is calculated using $128.1 + 37.6 \log_{10} d$ with distance d (in Kilometers) [38], and the radius of the cell is set to 500 m. The drain efficiency of the power amplifier ζ is set to 38% in our simulation whilst the energy harvesting efficiency is set to $\eta = 50\%$. The power budget for each BS is considered to be 46 dBm and the circuit power P_C is 40 dBm. The minimum harvesting energy E_k is set to 10% of the maximum transmit power. It is noted that these system parameters are merely chosen to demonstrate the EE optimization in an example and can easily be modified to any other values to address different scenarios.

In the first simulation, the performance of the proposed extended BC-MAC duality-based EE maximization algorithm is studied. The convergence behavior of the proposed Dinkelbach method-based scheme and quasi-concavity-based approach are first evaluated by illustrating how the EE performance behaves with the number of iterations. As shown in Fig. 2, both the proposed Dinkelbach method-based iterative resource allocation scheme and the proposed quasi-concavity-based scheme converge to the optimal value. In particular, EE converges after approximately 25 iterations when $\varsigma = 2$. However, the convergence procedure reduces to 10 iterations when a larger step size has been chosen, i.e., $\varsigma = 5$. This result coincides with our theoretical findings where the computational complexity is inversely proportional to the square of the step size ς^2. Hence, compared to the proposed Dinkelbach method-based scheme, the proposed quasi-concavity-based scheme is more efficient when an appropriate ς has been chosen. We then evaluate the EE to transmission power relationship for the case. It can be seen from Fig. 3 that the EE-transmission power relationship has a bell shape curve and is quasi-concave. This quasi-concavity is the foundation of the proposed methodology and infers that the proposed quasi-concavity-based resource allocation algorithm always leads to the maximum EE performance. In addition, the convergence behavior of the proposed upper bound sub-gradient resource allocation algorithm is also studied. Fig. 4 plots the EE versus the number of iterations for step sizes 0.1 and 0.01. As can be seen from the figure, the proposed extended BC-MAC duality-based EE maximization algorithm converges to a stable value, and the step size affects the accuracy and convergence speed of the algorithm.

In the next simulation, the proposed extended BC-MAC duality-based EE maximization algorithm under different maximum transmit power allowance is evaluated and presented in Fig. 5. To show the EE gain achieved by TS-based SWIPT system, we compare our proposed scheme with the scheme that maximize the EE without EH [17] and the scheme that aims for maximizing the system sum rate [39]. It is observed that the EE achieved by our proposed extended BC-MAC duality-based EE maximization algorithm is monotonically non-decreasing with respect to the maximum transmit power allowance P_{max}. Particularly, the EE is increasing dramatically with an increasing maximum transmit power allowance P_{max} in the higher transmit power constraint region, i.e., $P_{max} > 25$ dBm. This is because in the higher transmit power constraint region, a balance between the system EE and the total power consumption can be achieved. On the other hand, all the algorithms achieve similar performance in terms of the system EE criterion in the lower transmit power constraint region, i.e., $5 < P_{max} < 15$ dBm. Besides, due to the fact that the received power of the desired signal may not be sufficiently large for delivering information and energy harvesting at the same time, the system with TS-based energy harvesting receivers achieves a small performance gain compared to the system without energy harvesting receivers. Nevertheless, in the region of higher transmit power, the proposed extended BC-MAC duality-based EE maximization algorithm outperforms the other two schemes substantially. In particular, there is about a 5% gain in terms of EE can be achieved by our proposed extended BC-MAC duality-based EE maximization algorithm compared to the scheme that without energy harvesting receivers [17]. Furthermore, due to the fact that the increasing sum rate of the system cannot offset the consumption of the transmit power, the sum rate maximization scheme without energy harvesting [39] achieves a very low EE.

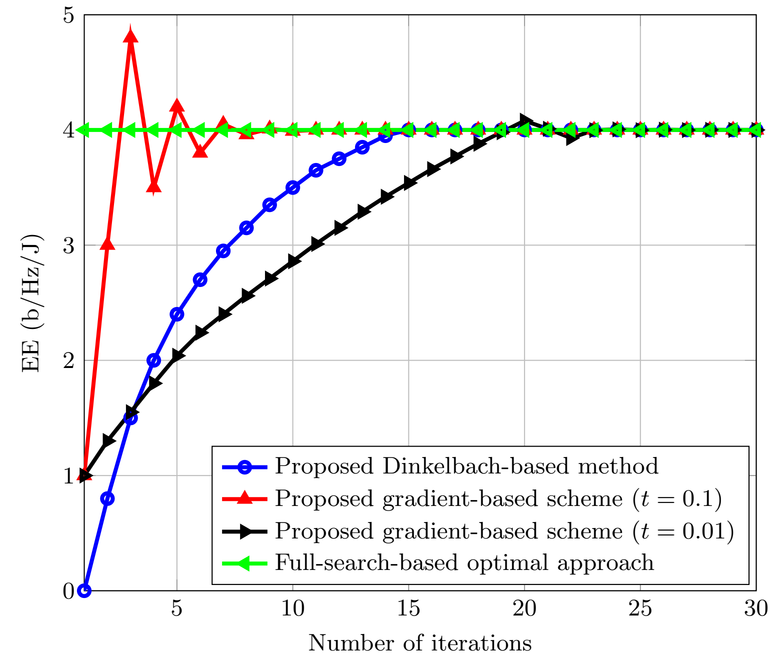

Fig. 2. Convergence behavior of the proposed Dinkelbach method-based scheme and quasi-concavity-based approach in terms of EE.

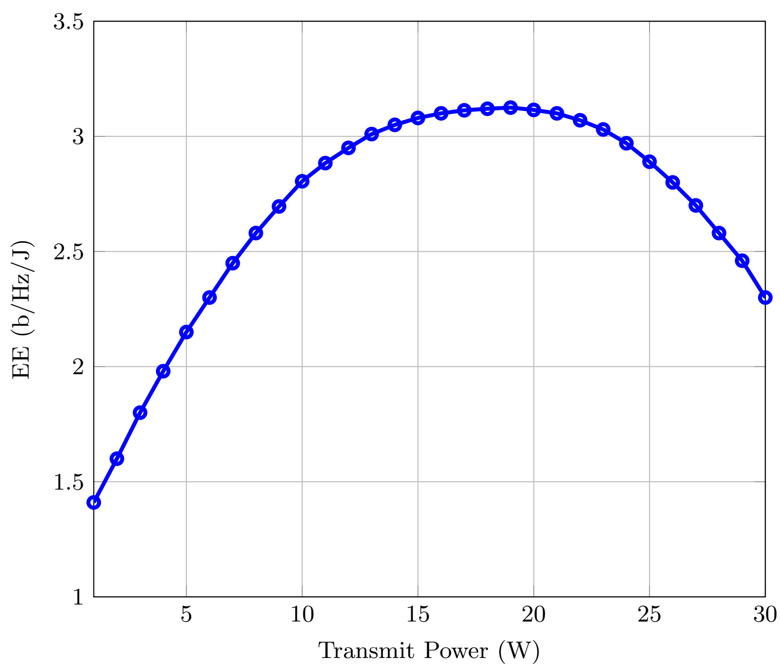

Fig. 3. λ_{EE}^{*}-versus-P_T^m curve using the proposed quasi-concavity-based resource allocation approach.

Finally, we investigate the average system EE versus the maximum transmit power allowance for the proposed extended BC-MAC duality-based EE maximization algorithm with different level of minimum required harvested energy. As shown in Fig. 6, the increasing level of minimum required harvested energy will not always lead to an increasing system EE. Furthermore, jointly considering the results in Fig. 6 and Fig. 5, we can conclude that there exists an optimal minimum required harvested energy value for the EE optimization problem. As a result, the performance of system EE can be further improved if the TS ratios and the minimum required harvested energy are jointly considered, and that would be investigated in our future works.

VIII. Conclusions

In this paper, we addressed the EE optimization problem for SWIPT-based MIMO-BC with TS receiver. Our aim was to maximize the EE of the system whilst satisfying constraints in terms of maximum power and minimum harvested energy for

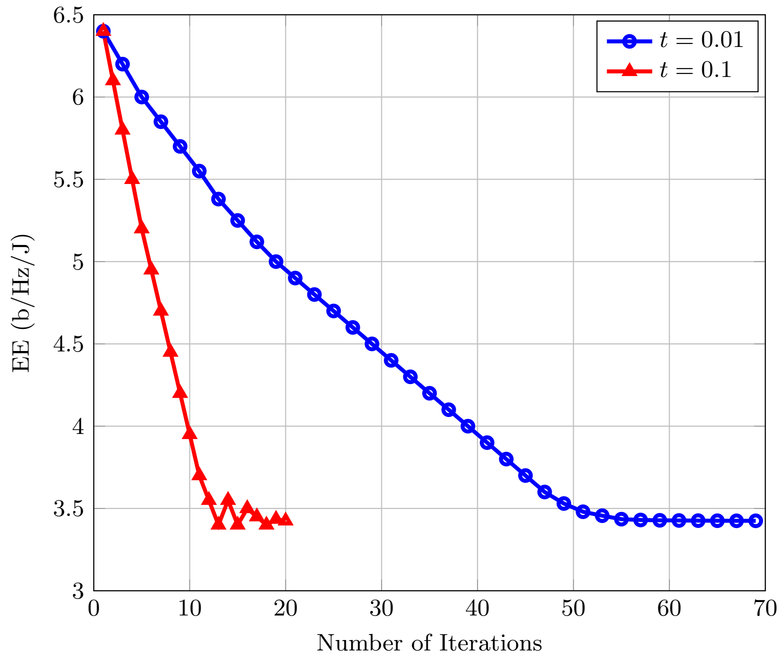

Fig. 4. Convergence behavior of the proposed extended BC-MAC duality-based EE maximization algorithm in terms of EE.

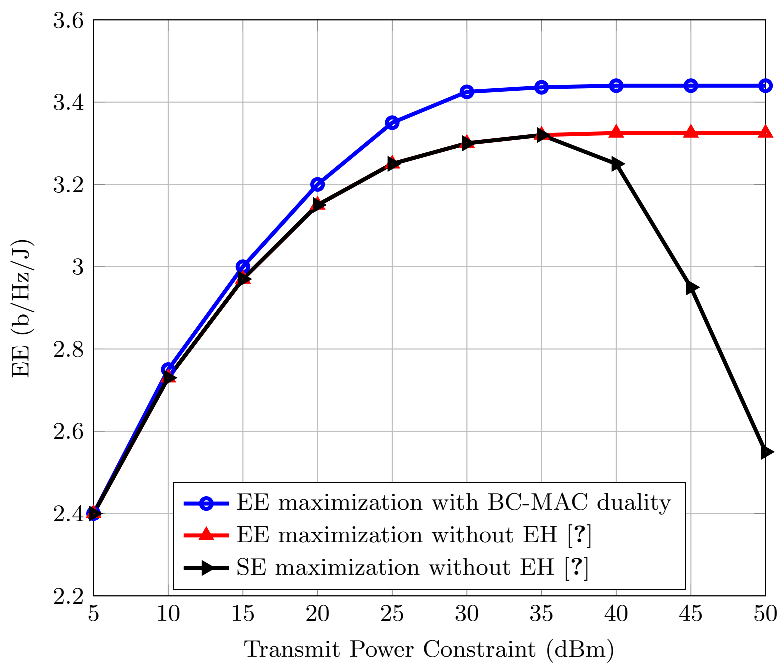

Fig. 5. The performance of the proposed extended BC-MAC duality-based EE maximization algorithm (EE vs transmit power constraint).

each user. The corresponding EE maximization problem from the coupling of the optimization variables, namely the transmit covariance matrices and TS ratios, is non-convex. Hence, to tackle the problem, we transform the original maximization problem with multiple constraints into a min-max problem with single constraint and multiple auxiliary variables. For the min-max problem with single constraint, a dual-layer resource allocation strategy is proposed. We incorporate an extended SWIPT-based BC-MAC duality principle in order to simplify the inner-layer problem, and accordingly provide two different iterative resource allocation algorithms for solving the dual MAC problem with fixed auxiliary variables. A sub-gradient-based searching scheme is then proposed to obtain the optimal auxiliary variables in the outer-layer. Numerical results validate the effectiveness of the proposed algorithms and show that significant performance gain in terms of EE can be achieved by our proposed extended BC-MAC duality-based EE maximization algorithm.

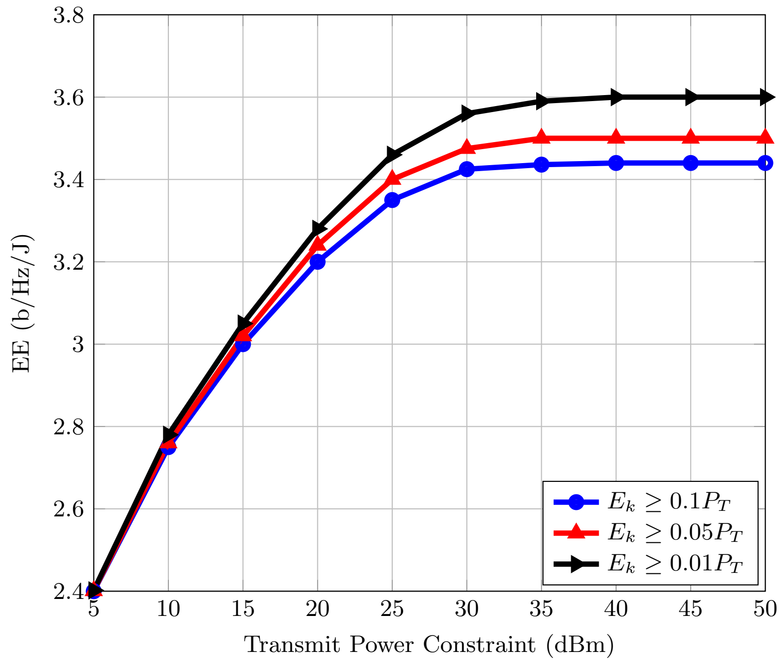

Fig. 6. Energy efficiency versus the maximum transmit power allowance for the proposed extended BC-MAC duality-based EE maximization algorithm with different minimum required power transfer.

Appendix A
Proof of Proposition 1

Suppose $\mathbf{Q}_k^b$ is a feasible solution for the problem in (9)-(12), it is clearly that $\mathbf{Q}_k^b$ is also feasible for the problem in (13)-(14). Thus, the feasible region of the problem (9)-(12) is a subset of that of the problem (13)-(14). Next, we need to prove the relationship is a tight upper bound. Specifically, the optimal solution of the optimization problem in (9)-(12) achieves the upper bound $g(\chi, \boldsymbol{\mu})$ for some χ and $\boldsymbol{\mu}$, where $\boldsymbol{\mu} = [\mu_1 \ \mu_2 \ \cdots \ \mu_K]$, and $g(\chi, \boldsymbol{\mu}) = \max_{\mathbf{Q}_k^b \succeq 0, 0 \leq \alpha_k \leq 1} \frac{\sum_{k=1}^K \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^K \sum_{k=1}^K (1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k \mathbf{Q})}$.

Consequently, we can list the Karush-Kuhn-Tucker (KKT) condition of the problem (9)-(12) as follows

$$\frac{\partial \lambda_{EE}}{\partial \mathbf{Q}_k^b} = \varrho \mathbf{I} - \sum_{k=1}^K \rho_k (1-\alpha_k)\eta_k \mathbf{G}_k + \mathbf{\Omega}_k, \ \forall k \in \mathcal{K} \tag{53}$$

$$\frac{\partial \lambda_{EE}}{\partial \alpha_k} = \rho_k \eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) + \theta_k, \ \forall k \in \mathcal{K} \tag{54}$$

$$\varrho(\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^b) - P_{\text{max}}) = 0 \tag{55}$$

$$\rho_k (E_{k,min} - (1-\alpha_k)\eta_k [\text{tr}(\mathbf{G}_k \mathbf{Q})]) = 0, \ \forall k \in \mathcal{K} \tag{56}$$

where ϱ and ρ_k are the Lagrange multipliers with respect to the two constraints, respectively, and $\mathbf{\Omega}_k$ and θ_k are the Lagrange multiplier associated with the constraint $\mathbf{Q}_k \geq 0$ and $0 \leq \alpha_k \leq 1$. Suppose the optimal solution of the optimization problem in (9)-(12) is obtained, and the corresponding optimal auxiliary variables are $\mathbf{Q}_k^*$, α_k^*, ϱ^* and ρ_k^*, $\mathbf{W}_k^*$ and $\mathbf{Q}_k^*$, $\forall k \in \mathcal{K}$, we can list the KKT condition of the problem (13)-(14) as follows

$$\frac{\partial \lambda_{EE}}{\partial \mathbf{Q}_k^b} = \varsigma \left(\bar{\varrho} \mathbf{I} - \sum_{k=1}^K \bar{\rho}_k (1-\alpha_k)\eta_k \mathbf{G}_k \right) + \mathbf{\Psi}_k, \ \forall k \in \mathcal{K} \tag{57}$$

$$\frac{\partial \lambda_{EE}}{\partial \alpha_k} = \varsigma \rho_k \eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) + \theta_k, \ \forall k \in \mathcal{K} \tag{58}$$

$$\varsigma \left\{ \bar{\varrho} \left(\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^b) - P_{\text{max}} \right) + \sum_{k=1}^K \bar{\rho}_k (E_{k,min} - (1-\alpha_k)\eta_k [\text{tr}(\mathbf{G}_k \mathbf{Q})]) \right\} = 0, \tag{59}$$

where ς and $\mathbf{\Psi}_k$ are the Lagrange multipliers associated with the constraint (14) and the constraint $\mathbf{Q}_k \geq 0$, respectively. If we set $\mathbf{Q}_k = \mathbf{Q}_k^*$, $\alpha_k = \alpha_k^*$, $\varsigma = 1$, $\bar{\varrho} = \varrho^*$ and $\bar{\rho}_k = \rho_k^*$, $\mathbf{\Psi}_k = \mathbf{W}_k^*$ and $\theta_k = \theta_k^*$, $\forall k \in \mathcal{K}$, the KKT conditions (57)-(59) are

met, meaning the upper-bound is tight. ■

APPENDIX B
PROOF OF PROPOSITION 2

We can rewrite the constraint (16) (left hand side) as the following linear formulation

$$\begin{aligned}&\chi\sum_{k=1}^{K}\text{tr}(\mathbf{Q}_k^b)-\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k[\text{tr}(\mathbf{G}_k\mathbf{Q})]\\&=\chi\text{tr}(\sum_{k=1}^{K}\mathbf{Q}_k^b)-\text{tr}(\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k\mathbf{G}_k\mathbf{Q}))\\&=\chi\text{tr}(\mathbf{Q})-\text{tr}(\mathbf{G}\mathbf{Q}))\end{aligned}\tag{60}$$

where $\mathbf{G}=\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k\mathbf{G}_k$. It is clear that constraint (16) satisfies the general linear power constraint $\text{tr}(\mathbf{AQ})\leq P$. Furthermore, it has been shown in [35] that the weighted factors are shared between the MAC and the BC, thus the TS ratios $\boldsymbol{\alpha}$ are sharing by the MAC and BC. On the other hand, if there exists another solution set $\breve{\boldsymbol{\alpha}}$ that maximizes the MAC problem but different from the optimal TS ratio $\boldsymbol{\alpha}$ in the BC, we can also obtain another optimal covariance matrices set in dual MAC. This is because the noise covariance at the BS in the dual MAC is written as $\boldsymbol{N}=\chi\mathbf{I}-\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k[\text{tr}(\mathbf{G}_k\mathbf{Q})])$. Apparently changing the TS factors in the dual MAC problem has impact on determining the optimal transmit covariance matrices. Therefore, this contradicts the uniqueness of the covariance mapping from the dual MAC to BC. As a result, we can infer that the duality relationship between SWIPT-based MIMO-BC system and its dual SWIPT-based MIMO-MAC stills holds. ■

APPENDIX C
PROOF OF PROPOSITION 3

According to *Proposition 2*, the capacity region of a SWIPT-based MIMO-BC with power constraint P_{all} is equal to the union of capacity regions of the SWIPT-based dual-MAC with power constraints such that $\sum_{k=1}^{K}\text{tr}(\mathbf{Q}_k^m)=P_{all}$. Substitute the duality result to problem (22)-(23), the objective function can be reformulated as

$$\max_{\mathbf{Q}_k^m\succeq 0,0\leq\alpha_k\leq 1}\ \frac{\sum_{k=1}^{K}\alpha_k R_k^c}{\zeta\chi\sum_{k=1}^{K}\text{tr}(\mathbf{Q}_k^b)-\zeta\sum_{k=1}^{K}\mu_k(1-\alpha_k)\eta_k\text{tr}(\mathbf{G}_k\mathbf{Q})+P_C}.\tag{61}$$

On the other hand, we can proceed by defining $g(\chi,\boldsymbol{\mu})$ for some χ and $\boldsymbol{\mu}$, where $\boldsymbol{\mu}=[\mu_1\ \mu_2\ \cdots\ \mu_K]$, and $g(\chi,\boldsymbol{\mu})=\max_{\mathbf{Q}_k^b\succeq 0,0\leq\alpha_k\leq 1}\frac{\sum_{k=1}^{K}\alpha_k R_k^b}{\zeta P_T^m+P_C-\sum_{k=1}^{K}\sum_{k=1}^{K}(1-\alpha_k)\eta_k\text{tr}(\mathbf{G}_k\mathbf{Q})}$. Hence, the objective function of problem (13)-(14) can be reformulated as

$$\min_{\chi\leq 0,\boldsymbol{\mu}\leq\mathbf{0}}\ g(\chi,\boldsymbol{\mu}).\tag{62}$$

Comparing (62) with (61), it can be observed that by choosing $\chi=1$ and $\boldsymbol{\mu}=\mathbf{1}$, these two optimization problems share the same solution set. Hence, the feasible region of the problem (61) is a subset of that of the problem (62). ■

APPENDIX D
PROOF OF LEMMA 1

Let ν denote the sub-gradient of $g(\check{\tau})$. For a given $\check{\tau}>0$, the sub-gradient ν of $g(\check{\tau})$ satisfies $g(\hat{\tau})\leq g(\check{\tau})+\nu(\hat{\tau}-\check{\tau})$, where $\hat{\tau}$ is any feasible value. Let $\hat{\mathbf{Q}}_k^m,\{k=1,\cdots,K\}$ and $\hat{\alpha}_k,\{k=1,\cdots,K\}$, respectively denote the optimal covariance matrices and TS ratio factors in (33) for $\tau=\hat{\tau}$, and $\check{\mathbf{Q}}_k^m,\{k=1,\cdots,K\}$ and $\check{\alpha}_k,\{k=1,\cdots,K\}$ represent the optimal counterparts. Define $h(\mathbf{Q}_1^m,\cdots,\mathbf{Q}_K^m,\alpha_1,\cdots,\alpha_K)=\Delta_k\log|\mathbf{N}+\sum_{k=1}^{K}\mathbf{H}_k^H\mathbf{Q}_k^m\mathbf{H}_k|-\beta(\zeta\sum_{k=1}^{K}\text{tr}(\mathbf{Q}_k^m)+P_C)$, we formulate $g(\hat{\tau})$ as follows

$$\begin{aligned}g(\hat{\tau})&=\max_{\mathbf{Q}_k^m,\alpha_k}\ h(\mathbf{Q}_1^m,\cdots,\mathbf{Q}_K^m,\alpha_1,\cdots,\alpha_K)-\hat{\tau}[P_{all}-\sum_{k=1}^{K}\text{tr}(\mathbf{Q}_k^m)]\\&=h(\hat{\mathbf{Q}}_1^m,\cdots,\hat{\mathbf{Q}}_K^m,\hat{\alpha}_1,\cdots,\hat{\alpha}_K)-\hat{\tau}[P_{all}-\sum_{k=1}^{K}\text{tr}(\hat{\mathbf{Q}}_k^m)]\\&\geq h(\check{\mathbf{Q}}_1,\cdots,\check{\mathbf{Q}}_K^m,\check{\alpha}_1,\cdots,\check{\alpha}_K)-\hat{\tau}[P_{all}-\sum_{k=1}^{K}\text{tr}(\check{\mathbf{Q}}_k^m)]\end{aligned}$$

$$= h(\check{\mathbf{Q}}_1^m, \cdots, \check{\mathbf{Q}}_K^m, \check{\alpha}_1, \cdots, \check{\alpha}_K) - \check{\tau}[P_{all} - \sum_{k=1}^{K} \mathrm{tr}(\check{\mathbf{Q}}_k^m)]$$

$$+ \check{\tau}[P_{all} - \sum_{k=1}^{K} \mathrm{tr}(\check{\mathbf{Q}}_k^m)] - \hat{\tau}[P_{all} - \sum_{k=1}^{K} \mathrm{tr}(\check{\mathbf{Q}}_k^m)]$$

$$= g(\check{\tau}) + (\hat{\tau} - \check{\tau})[P_{all} - \sum_{k=1}^{K} \mathrm{tr}(\check{\mathbf{Q}}_k^m)]. \tag{63}$$

Hence the sub-gradient of $g(\check{\tau})$ is $P_{all} - \sum_{k=1}^{K} \mathrm{tr}(\check{\mathbf{Q}}_k^m)$. ■

Appendix E
Proof of Proposition 5

$\vartheta(\alpha)$ can be reformulated as $\vartheta(\alpha) = \alpha\varphi(\alpha)$, where $\varphi(\alpha) = \sum_{k=1}^{K} R_k^m$. As shown in [40], the second-order conditions for concave function is shown as

$$\nabla^2 \vartheta(\alpha) \leq 0. \tag{64}$$

Therefore, we express $\nabla^2 \vartheta(\alpha)$ as

$$\nabla^2 \vartheta(\alpha) = \nabla_\alpha(\varphi(\alpha) + \alpha \nabla_\alpha \varphi(\alpha)).$$

$$= 2\nabla_\alpha \varphi(\alpha) + \alpha \nabla_\alpha^2 \varphi(\alpha). \tag{65}$$

Since $\varphi(\alpha)$ is concave in α, $\nabla_\alpha^2 \varphi(\alpha) \leq 0$. Furthermore, as the noise covariance is written as $\mathbf{N} = \chi \mathbf{I} - \sum_{k=1}^{K} \mu_k (1 - \alpha)\eta_k[\mathrm{tr}(\mathbf{G}_k \mathbf{Q})])$, increasing α will enhance the noise variance and hence reduce the corresponding rate. Thus, $\nabla_\alpha \varphi(\alpha) \leq 0$. ■

Appendix F
Proof of Proposition 6

To prove $\lambda_{EE}^*(P_T^m)$ is a quasi-concave function, we denote the superlevel sets of $\lambda_{EE}^*(P_T^m)$ as

$$\mathcal{S}_\kappa = \{P_T^m \leq P_{all} | \lambda_{EE}^*(P_T^m) \geq \kappa\}. \tag{66}$$

In accordance with [40], for any real number κ, if the convexity for $\mathcal{S}_\kappa$ holds, $\lambda_{EE}^*(P_T^m)$ is strictly quasi-concave in P_T^m. Therefore, we here divide the proof into two cases. For the case of $\kappa < 0$, since EE is always positive and hence there are no points on the counter, $\lambda_{EE}^*(P_T^m) = \kappa$. For the case of $\kappa \geq 0$, λ_{EE} can be rewritten as

$$\lambda_{EE} = \frac{C_{MAC}(P_T^m)}{\zeta P_T^m + P_C}, \tag{67}$$

and hence $\mathcal{S}_\kappa$ is equivalent to $\kappa\zeta P_T^m + \kappa P_C - C_{MAC}(P_T^m) \leq 0$. Since it has been proven that $C_{MAC}(P_T^m)$ is convex in P_T^m [23], therefore the convexity of $\mathcal{S}_\kappa$ holds and $\lambda_{EE}^*(P_T^m)$ is strictly quasi-concave in P_T^m. ■

Appendix G
Proof of Proposition 7

In order to provide a proof for the statement in *Proposition 7*, we analyze the limit of $\lambda_{EE}^*(P_T^m)$ as in the following expression

$$\lim_{P_T^m \to 0} \lambda_{EE}^*(P_T^m) = \lim_{P_T^m \to 0} \max_{\mathbf{Q}_k^m \succeq 0,\ 0 \leq \alpha_k \leq 1} \frac{C_{MAC}(P_T^m)}{\zeta P_T^m + P_C}.$$

$$= \lim_{P_T^m \to 0} \frac{o(P_T^m)}{\zeta P_T^m + P_C} = 0. \tag{68}$$

Thus, given that $\lambda_{EE}^*(P_T^m)$ is strict concave (Appendix F), beginning with $P_T^m = P_{min}$, $\lambda_{EE}^*(P_T^m)$ either strictly decreases with P_T^m while $\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} \leq 0$, or first strictly increases and then strictly decreases with P_T^m while $\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} > 0$. In addition, we can conclude that the maximum EE achieved in the power region $[P_{min}, P_{max}]$ is straightforward as indicated in *Proposition 7*. ■

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